ISSN 2229-5518

Inverse Kinematic Scheme to Organize Unsteady Flow in Open Channels

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Abstract—Water scarcity in Egypt has undoubtedly grown to become a serious problem. Mismanagement of water resources, ineffective irrigation techniques, and inexact operation and management of irrigation canals are some factors leads to water waste in Egypt. The adjustment of hydraulic structures' gates at intakes of irrigation canals leads to transitional unsteady flow at beginning of operation until reaches another steady flow. Unsteady open channels flow can be simulated using routing problems and operation type problems. St.Venant equations are used in routing schemes to find the discharge and water level above certain datum along the channel in future time. In operation-type problems, the same equations are used to determine the inflow hydrographs at channel's intakes which achieve the prescribed hydrographs at the end section of channel. Operation-type problems are also referred to as inverse calculation of unsteady open channels flow. A new inverse kinematic scheme is presented herein to control unsteady flow in open channel. Momentum equation and continuity equation for kinematic wave are combined to derive a kinematic wave equation with discharge *Q* as dependent variable. The inverse kinematic algorithm was applied to get a solution of kinematic wave equation based on the approximation of partial derivatives and coefficients of Preissmann scheme. The inflow hydrographs are obtained by moving in reverse in the direction of time at first and then backward in the direction of space. The calculated upstream hydrographs found using inverse kinematic scheme revealed identical accuracy as those obtained using inverse explicit scheme. To examine the accuracy of calculated inflow hydrograph by inverse kinematic method, it was used after that as upstream boundary condition for Verwy variant of Preissmann scheme. The calculated downstream hydrographs produced approximately the required water demand hydrograph. The mathematical model of Verwy variant of Preissmann scheme solves complete governing equations in routing problem.

Index Terms— Unsteady open channel flow, St. Venant equations, inverse kinematic scheme, inverse explicit scheme, Preissmann scheme, routing problems, Operation problems.

1 Introduction

Water uses in Egypt have exceeded the available resources due to rapid population growth, increased food demand, development and renewal of the industrial base, and enhanced standards of living.

Ethiopian Renaissance Dam may decrease Egypt's share of Nile water. So, Egyptians must rationalize their water usage to overcome water shortage problems in the future.

Barrages on river and regulators on canals have gates to organize the volume of flow released through their structures. Accurate water management in irrigation systems is important to maximize benefits to water consumers, minimizing waste of water, and expecting variation in water demand due to weather conditions and agriculture requirements.

Precise organize of water flow in canals became essential because of increasing water demands. Operation on irrigation canals aims to deliver fair water to users. Numerical models were developed for operation-type problems [2}, [6], [7], [10], [11], [13], [14], [15] to estimate the upstream inflow which yields the required downstream hydrograph.

A transitional gate stroking control technique [2],[16], utilized to organize gate movements and water released to provide water demand at downstream. The gate stroking technique uses more complicated characteristics method. The finite difference techniques provide easier solution to the governing equations of unsteady open channel flow.

The finite difference schemes are divided into explicit numerical schemes and implicit numerical schemes [1],[3],[5],[8],[12]. A rectangular time-space grid is used in explicit numerical schemes. The unknown discharges and water depths at grid points on future time are calculated using known discharges, water depths and conditions on current time line or on both current and previous time lines. Implicit type numerical schemes calculate the unknown discharges and water depths at advanced time levels on time-space rectangular grid by preparing a number of equations equal to the number of unknown variables and solving these equations simultaneously.

Preissmann type implicit schemes are commonly used to solve St. Venant equations. Large time steps can be used in implicit schemes due to its stability, but time step may be restricted to attain more accurate results. On the contrary of explicit numerical schemes, the time step in implicit schemes is not constrained by Courant condition [1],[3],[5],[8].

Inverse implicit scheme and inverse explicit scheme are the finite difference schemes that recently used to simulate the

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inverse problem. The implicit scheme use grid rotated 90° in the space-time grid [2]. In this implicit scheme, the prescribed discharges and depths at end section of channel are utilized as initial condition while the known variables at initial computational time and final computational time can be water depth profile or discharge profile.

Inverse explicit scheme mentioned in Liu, et al. 1992 [7] is explicit scheme use the discretization of Preissmann scheme. The computation in this explicit scheme initiates at top-right junction of time-space plane. The values of unknowns are found by moving in reverse in the direction of space at first and then in reverse in the direction of time. The problem of the inverse explicit scheme can be solved by moving in reverse in the direction of time at first and after that in reverse in the direction of space [14].

A new inverse kinematic scheme is explained in this research. The scheme is finite difference scheme using the approximation of partial derivatives and coefficient of Preissmann type scheme to calculate the inflow hydrographs. Specified flow hydrograph at channel's end is used as initial conditions in this scheme. The computed results using this scheme exhibited more stability than that calculated using inverse explicit scheme. Verwy variant of Preissmann implicit scheme was used after that to calculate the downstream hydrographs. In this scheme, the calculated upstream discharges profile using inverse kinematic scheme was utilized as upstream condition. The calculated hydrographs at end section using Verwy variant of Preissmann scheme approximately reproduced the required hydrograph.

2 One Dimensional Saint Venant Equations

St. Venant equations in conservation and non-conservation forms are used to simulate various one-dimensional routing models. Assuming no lateral outflow, St. Venant equations are written as:

Continuity equation

Conservation form
$$b\frac{\partial y}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

Non-conservation form
$$V\frac{\partial y}{\partial x}+y\frac{\partial V}{\partial x}+\frac{\partial y}{\partial t}=0 \tag{2}$$

Momentum equation

Conservation form

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\beta\frac{Q^2}{A}\right)+gA\left(\frac{\partial y}{\partial x}+S_f-S_0\right)=0 \tag{3}$$

Non-conservation form (unit width element)

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial y}{\partial x}-g(S_f-S_0)=0 \tag{4}$$

where: A = wetted cross section area; b = top width of wetted area; g = gravitational acceleration; Q = discharge through cross section area; y = flow depth; V = velocity of flow; t = time; x = space; S_0= channel bed slope and S_f = friction slope.

Alternative routing models of flow are simulated by using full continuity equation with all or some terms of momentum equation. The mathematical models of dynamic wave take acceleration and pressure terms in momentum equation into account, while the mathematical models of kinematic wave ignore these terms. Kinematic wave scheme assumes S_0= S_f and the friction forces balance the gravity forces [3], [4], [9].

Continuity and momentum equations in the kinematic wave scheme are written as:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{5}$$

$$S_0 = S_f \tag{6}$$

Momentum equation (6) can be written as:

$$A=\alpha Q^m \tag{7}$$

Using Manning's equation, $m = 0.6$ and $\alpha=\left[\frac{nP^{2/3}}{\sqrt{S_0}}\right]^{0.6}$

To eliminate A from equation (5), differentiate equation (7) to get:

$$\frac{\partial A}{\partial t}=\alpha m Q^{m-1}\left(\frac{\partial Q}{\partial t}\right) \tag{8}$$

Then, substituting for $\frac{\partial A}{\partial t}$ in equation (5) leads to:

$$\frac{\partial Q}{\partial x}+\alpha m Q^{m-1}\left(\frac{\partial Q}{\partial t}\right)=0 \tag{9}$$

3 Implicit Type Preissmann Routing scheme

Preissmann implicit scheme is extensively used as a result of its simple structure at all grid points with flow and geometrical variables [1, 3, 4, and 7].

The grid of Preissmann scheme is illustrated in Fig.1. The derivatives in equations (1) and (2) can be substituted as:

$$\frac{\partial f}{\partial t}=\phi\frac{f_{i+1}^{k+1}-f_{i+1}^{k}}{\Delta t}+(1-\phi)\frac{f_i^{k+1}-f_i^{k}}{\Delta t} \tag{10}$$

$$\frac{\partial f}{\partial x} = \theta \frac{f_{i+1}^{k+1} - f_i^{k+1}}{\Delta x} + (1-\theta)\frac{f_{i+1}^{k} - f_i^{k}}{\Delta x} \qquad (11)$$

$$f(x,t) = \theta\left[\phi\, f_{i+1}^{k+1} + (1-\phi)\, f_i^{k+1}\right] + (1-\theta)\left[\phi\, f_{i+1}^{k} + (1-\phi)\, f_i^{k}\right] \qquad (12)$$

where $f_i^k = f(i\Delta x, k\Delta t)$; Δt = grid interval along t-axis; Δx = grid interval along x-axis; ϕ = weighting distribution coefficient related to space and θ = weighting distribution coefficient related to time, $0 \le \theta \le 1$.

Verwey [5, and12], used a different approximation for term $\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)$ and resistance term $\frac{Q\,|Q|}{\lambda^2}$ in equation (3), as follows:

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = \frac{1}{\Delta x}\left[\left(\frac{Q_{i+1}^{k}\, Q_{i+1}^{k+1}}{A_{i+1}^{k+1/2}}\right) - \left(\frac{Q_i^{k}\, Q_i^{k+1}}{A_i^{k+1/2}}\right)\right] \qquad (13)$$

$$\frac{Q\,|Q|}{\lambda^2} = 0.5\left[\left(\frac{\left|Q_{i+1}^{k}\right| Q_{i+1}^{k+1}}{(\lambda_{i+1}^{k+1/2})^2}\right) - \left(\frac{\left|Q_i^{k}\right| Q_i^{k+1}}{(\lambda_i^{k+1/2})^2}\right)\right] \qquad (14)$$

The superscripts *k+1/2* means that the function is computed between two time levels *kΔt* and *(k+1)Δt*.

In the above terms, the variables at time level *k* are known and the variables at time levels *k+1* are unknowns. Substituting the above terms in equations (1) and (3) produces:

$$L\, y_i^{k+1} + M\, Q_i^{k+1} + N\, y_{i+1}^{k+1} + O\, Q_{i+1}^{k+1} + W = 0 \qquad (15)$$

$$L'\, y_i^{k+1} + M'\, Q_i^{k+1} + N'\, y_{i+1}^{k+1} + O'\, Q_{i+1}^{k+1} + W' = 0 \qquad (16)$$

Coefficients *L, M, N, O, W, L′, M′, N′, O′,* and *W′* in equations (15) and (16) can be computed using known discharges and water depths at time k. Equations (15) and (16) create a linear algebraic system of equations containing four unknowns. There are *I* points on row *k+1*, and *I-1* meshes produces a number of equations equal *2(I-1)* to calculate *2I* unknowns. Two extra equations or values of two unknowns at Boundary make the number of equations equal to the number of unknowns. Any method for solution of linear algebraic equations can be applied to solve these equations. The double-sweep method [4, 7] is very effective method decreases the time required to solve the above system of equations.

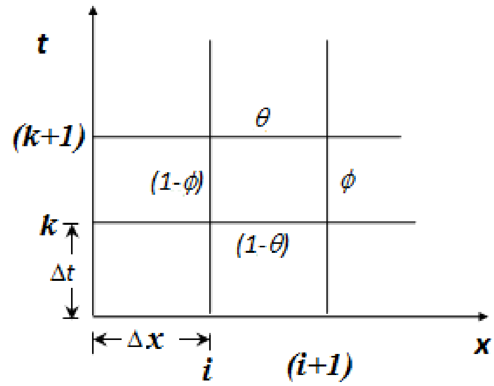

Fig. 1. Computational Grid for Verwy's variant of Preissmann Scheme.

4 Inverse Kinematic Scheme

The solution of the kinematic wave equation (9) can be established numerically using finite difference methods. For inverse kinematic scheme (operation type kinematic scheme), values of discharges and water depths at channel's end are specified. Inverse kinematic routing scheme based on the approximation of partial derivatives and coefficient of Preissmann scheme as mentioned in equations (17), (18), and (19). Specifying Q_i, and y_i between two consecutive time at downstream section, the discharge and water depth at time *(k-1)* is found by moving in reverse in time at first and after that in reverse in space as illustrated in Fig.2. In this scheme, derivatives in equation (9) are expressed as:

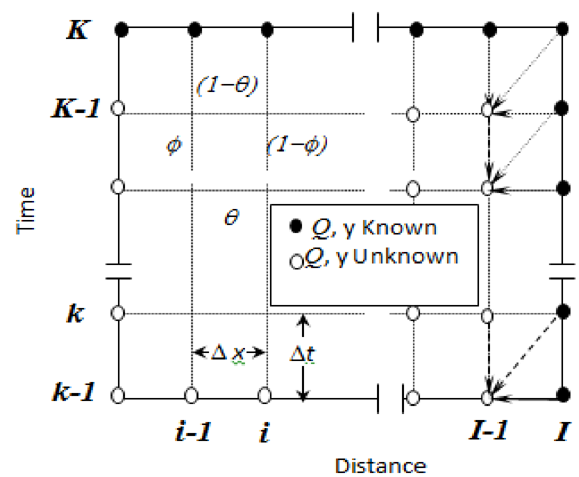

Fig. 2. Computational Grid for Inverse scheme.

A flow chart for inverse kinematic scheme is given in Fig. 3.

$$\frac{\partial f}{\partial t} = \phi \frac{f_{i-1}^{k} - f_{i-1}^{k-1}}{\Delta t} + (1-\phi)\frac{f_i^{k} - f_i^{k-1}}{\Delta t} \qquad (17)$$

ISSN 2229-5518

$$\frac{\partial f}{\partial x} = \theta \frac{f_i^{k-1} - f_{i-1}^{k-1}}{\Delta x} + (1-\theta)\frac{f_i^{k} - f_{i-1}^{k}}{\Delta x} \quad (18)$$

$$f(x,t) = \theta\left[\phi f_{i-1}^{k-1} + (1-\phi) f_i^{k-1}\right] + (1-\theta)\left[\phi f_{i-1}^{k} + (1-\phi) f_i^{k}\right] \quad (19)$$

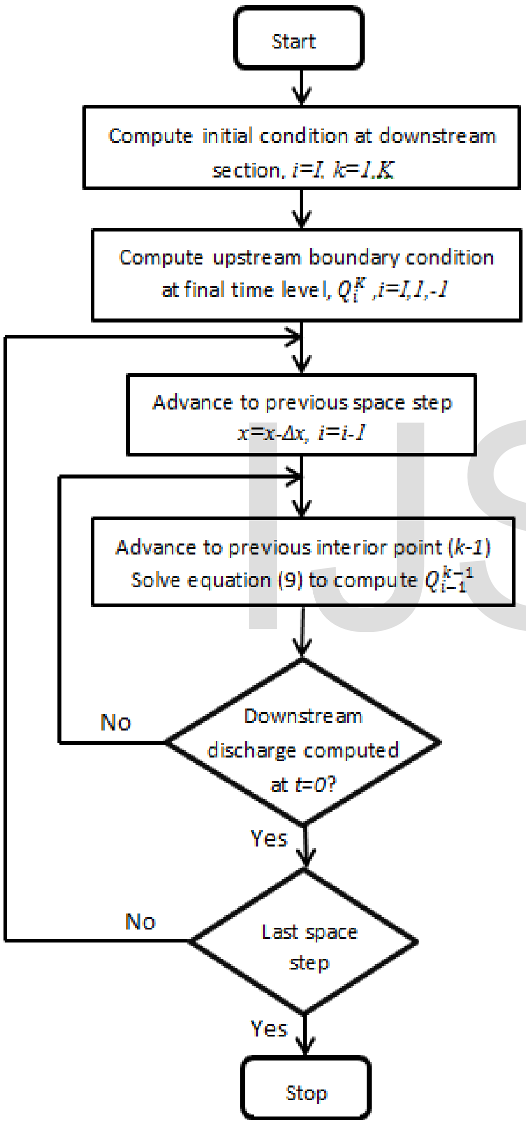

Fig. 3. Operation type kinematic scheme flow chart.

5 Inverse Explicit Scheme

For inverse (backward) explicit scheme (operation problems), discharges and water depths at end section are specified. The operation problem is an explicit finite difference method use the discretization of Preissmann scheme, equations (17), (18), and (19). The use of the finite difference approximations produces a system of two algebraic equations containing two unknowns. Knowing Q_i , and y_i between two consecutive time at downstream section, discharges and water depths at time *(K-1)* can be attained by moving firstly in reverse in direction of time and after that in reverse in direction of space as clarified in fig. 2..

6 Application of Inverse Kinematic Scheme

The validity of operation type kinematic scheme was investigated by example presented in Liu, et al. 1992 [7, 13, and 14]. The open channel in this example is trapezoidal channel with length 2.5 km, width 5.0m, longitudinal bottom slope equal 0.001, and side slopes 1.5H to 1V. The channel has Manning's roughness coefficient n equal to 0.025, and an overflow weir with free flow condition fixed at downstream end. The discharge rises at channel's end from 5.0m^3/s to10.0 m^3/s in one hour, and then the discharge remains constant at10 m^3/s for next two hours, after that reduced again to5.0 m3/s in one hour. The calculated discharge and water depth hydrographs at upstream section are illustrated in Figs. 4 and 5. These hydrographs were determined by using required discharges and water depths at channel's end as initial condition.

To simulate flow in this channel with Verwy's variant of Preissmann implicit scheme, the calculated discharge hydrograph at channel's intake was considered as upstream boundary condition and the weir equation fixed at downstream section was used to calculate downstream boundary condition. Using this implicit scheme, the calculated downstream hydrographs illustrated in Figs. 4 and 5 show good accuracy compared with the required hydrographs.

7 Numerical simulation and results analysis

Backward explicit scheme and operation type kinematic scheme were used separately to simulate the flow in the above described channel. The calculated discharge and depth hydrographs at channel intake are illustrated in Figs. 4 and 5. Space step Δx =100 m, time step Δt =100 s, weighting coefficient θ = 0.9, and weighting coefficient ϕ = 1.0 were utilized in operation type kinematic scheme, while Δx =100 m, Δt =600 s, weighting coefficient θ = 1.0 and weighting coefficient ϕ = 0.5 were utilized in backward explicit scheme. The calculated results using operation type kinematic scheme and backward explicit scheme indicate nearly similar accuracy.

The simulated discharge hydrograph computed at channel's intake by operation type kinematic scheme was utilized after that in Verwy's variant of Preissmann scheme as upstream boundary condition to computed hydrographs at channel's end. Figs. 4 and 5 indicate that the simulated downstream hydrographs were close to the required hydrographs.

The effect of space step Δx, time step Δt, weighting coefficient θ, and weighting coefficient ϕ on the performance of inverse kinematic scheme was tested.

The simulated upstream hydrographs using inverse kinematic scheme with Δx =100 m, 250 m, and 500 m are clarified in Figs. 6 and 7.The results indicates that the influence of space interval Δx on the simulated hydrographs is insignificant.

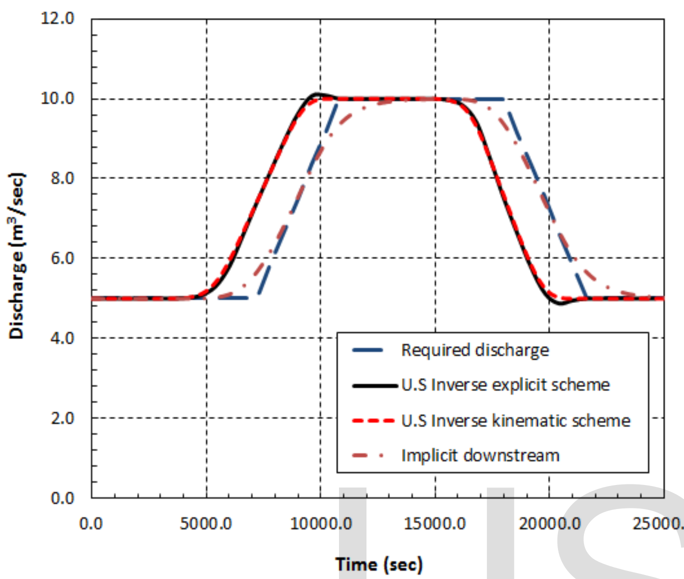

Fig.4. Simulated discharge hydrographs using operation type kinematic scheme and backward explicit scheme.

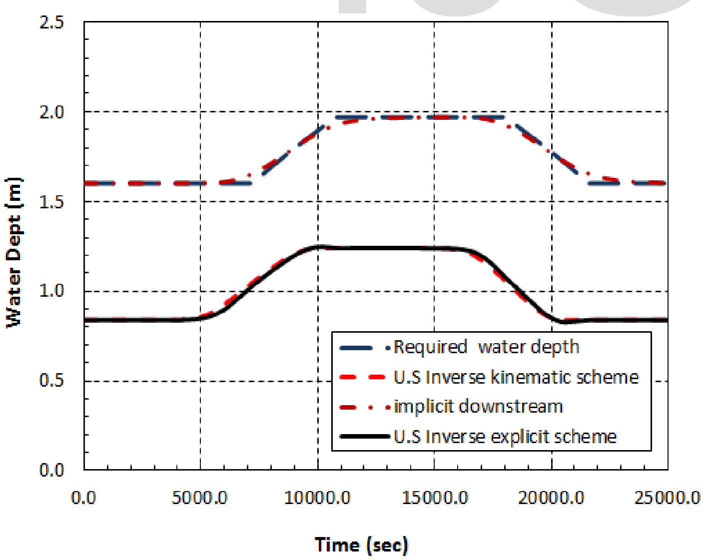

Fig.5. Simulated water depth hydrographs using operation type kinematic scheme and backward explicit scheme.

The simulated upstream hydrographs, using Δt =100 sec, 300 sec, and 600 sec are clarified in Figs 8 and 9. The result indicates that Δt has very small influence on the computed hydrographs.

The results attained using various values of weighting coefficient θ in inverse kinematic scheme are clarified in Figs. 10 and 11. The simulated upstream hydrograph using inverse kinematic scheme with θ = 0.9 show approximately similar accuracy as the computed upstream hydrograph attained using inverse explicit scheme.

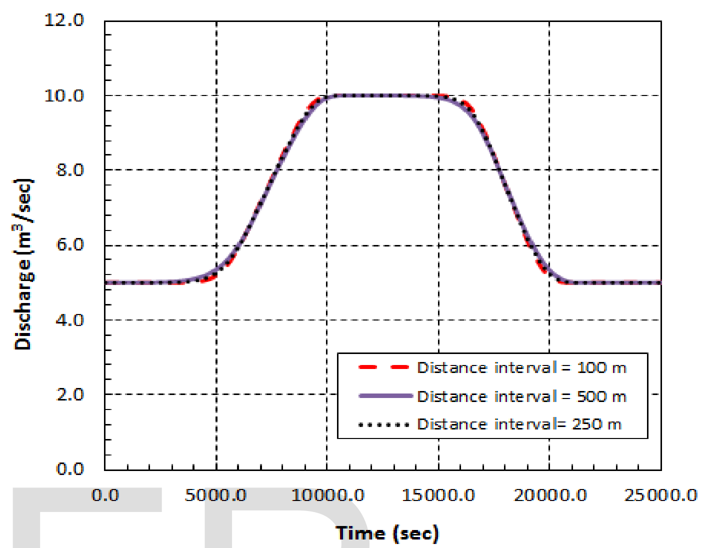

Fig.6. Simulated upstream discharge hydrographs using operation type kinematic scheme with different Δx.

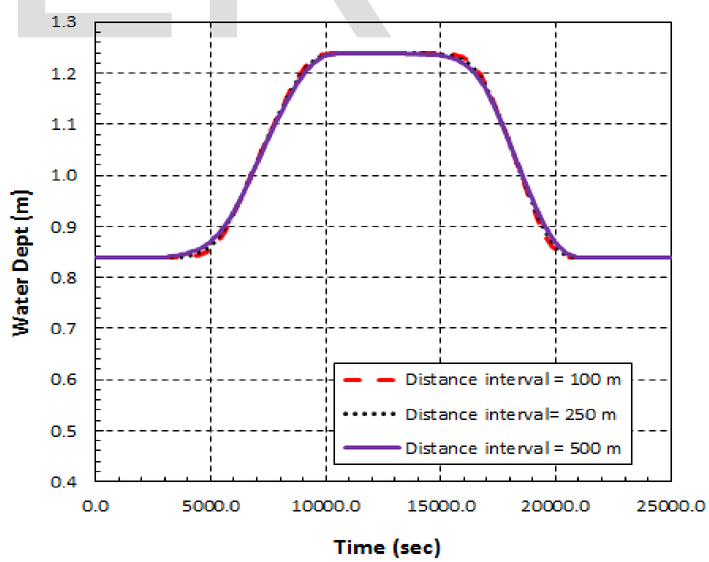

Fig.7. Simulated upstream water depth hydrographs using operation type kinematic scheme with different Δx.

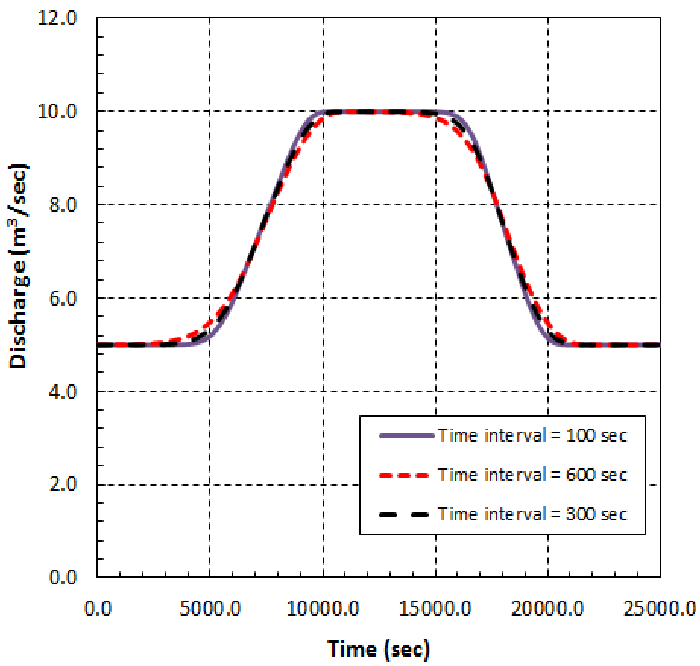

Fig.8. Simulated upstream discharge hydrographs using operation type kinematic scheme with different *Δt*.

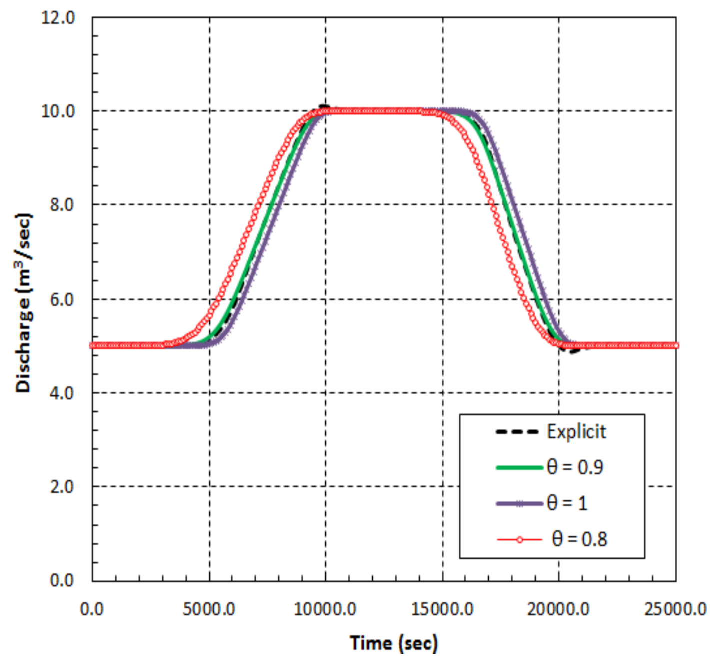

Fig.10. Simulated upstream discharge hydrographs using operation type kinematic scheme with different values of θ.

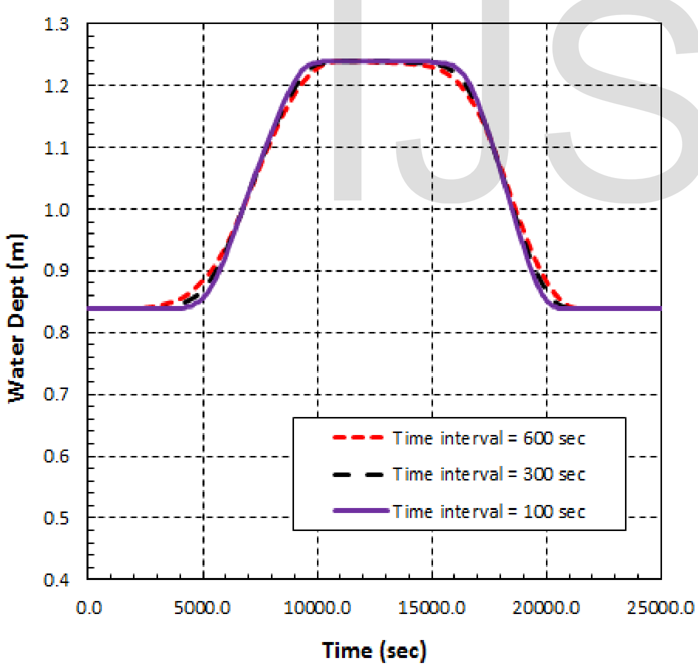

Fig.9. Simulated upstream water depth hydrographs using operation type kinematic scheme with different *Δt*.

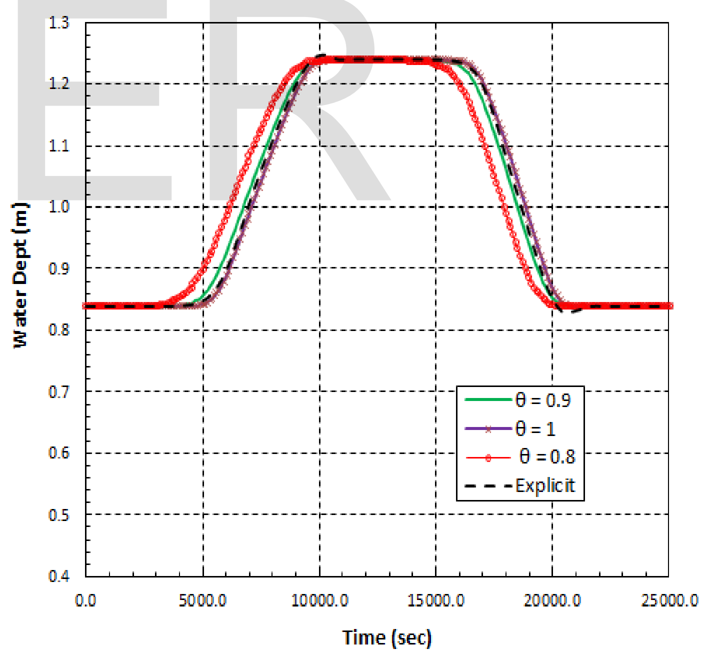

Fig.11. Simulated upstream water depth hydrographs using operation type kinematic scheme with different values of θ.

The simulated upstream hydrograph using inverse kinematic scheme with ϕ = 1.0 show approximately similar accuracy as that attained using inverse explicit scheme as illustrated in Figs. 12 and 13.

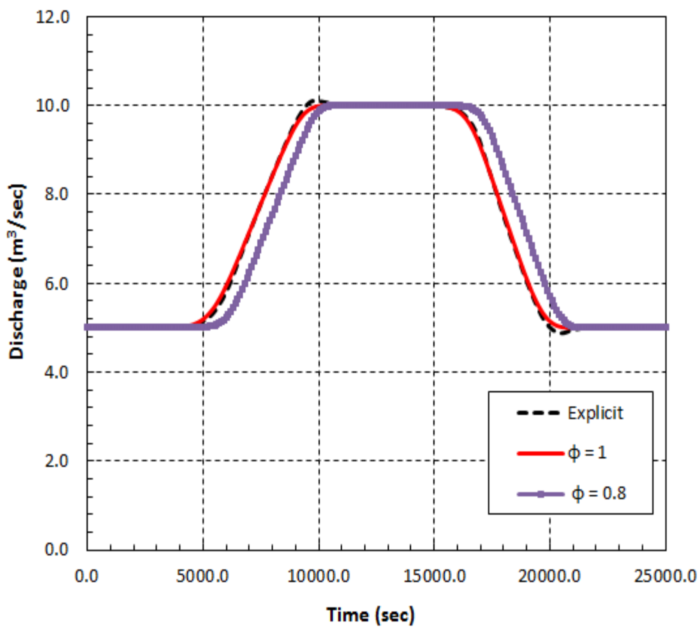

Fig.12. Simulated upstream discharge hydrographs using operation type kinematic scheme with different values of ϕ.

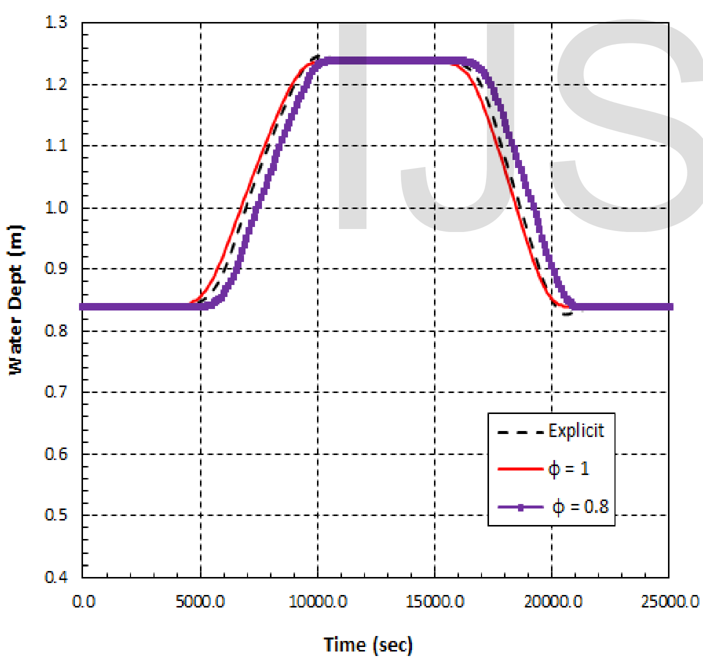

Fig.13. Simulated upstream water depth hydrographs using operation type kinematic scheme with different values of ϕ.

8 Conclusions

New finite difference inverse kinematic scheme has been presented herein to organize unsteady flow in open channels. The inverse kinematic scheme is numerically stable. This method compute the inflow hydrograph at canal' intake to achieve water demands at downstream end of canal.

The influence of space step Δx, time step Δt, weighting coefficient θ, and weighting coefficient ϕ on scheme performance was tested. The results show that space interval Δx has no effect on the simulated hydrographs. Also, the time interval Δt has very small effect on the simulated hydrographs. The simulated upstream hydrographs found using inverse kinematic scheme with θ = 0.9 and ϕ = 1.0 display good agreement with the simulated upstream hydrographs obtained using inverse explicit scheme with θ = 1.0 andϕ = 0.5. The attained hydrograph at canal's intake using the inverse kinematic scheme was used after that as upstream condition in Verwy's variant of Preissmann implicit scheme. The produced discharge and water depth hydrographs at channel's end using this scheme were very close to the required hydrographs.

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NOTATION

The following symbols are used in this paper:

A = wetted cross-sectional area;
b = wetted top width;
f = general function;
g = gravitational constant;
i = cross-section index;
k = time-level index;
n = Mannings roughness coefficient;
Q = discharge (through A);
P = wetted perimeter of cross section;
S_0 = bottom slope of the channel;
S_f = friction slope;
t = time;
V = velocity of flow;
x = space;
y = depth of flow ;
Δt = time step;
Δx = space step;
ϕ = a weighting coefficient for distributing terms in space; and
θ = a weighting coefficient for distributing terms in time.